PEWA: Patch-Based Exponentially Weighted Aggregation for Image Denoising

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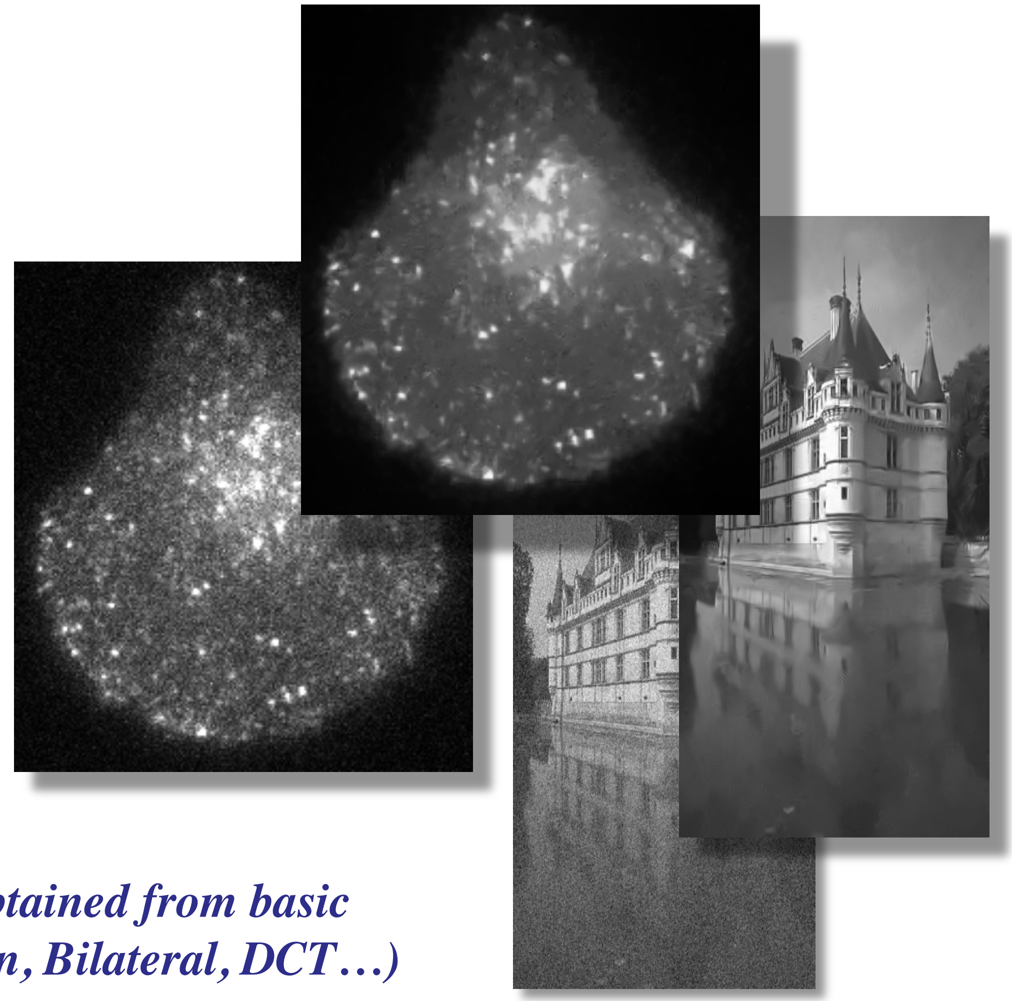
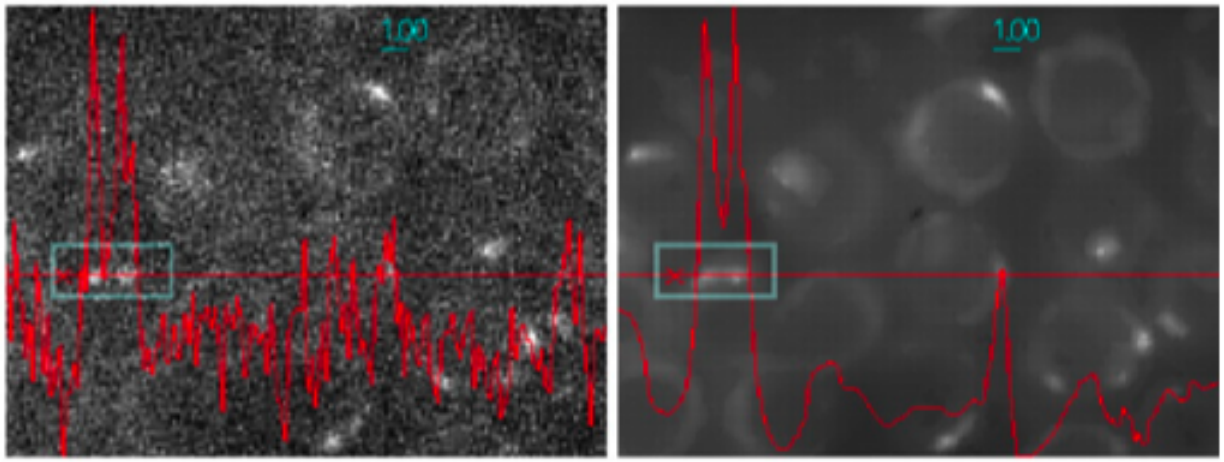


"Combining several restored images obtained from basic denoisers (Gaussian, Wiener, Median, Bilateral, DCT…) to get a boosted solution… "

Image Denoising: Motivation and Definition



An academic problem: recover $f : \mathcal{X} \subset \mathbb{R}^d \to \mathbb{R}_+$ from noisy data $\{v(x), x \in \mathcal{X}\}$:

$$v(x) = f(x) + \varepsilon(x) \quad \text{with} \quad \mathbb{E}[\varepsilon(x)] = 0, \quad \mathrm{Var}[\varepsilon(x)] = \sigma^2$$

... discontinuities, textures, homogeneous regions and image organization must be preserved !

Competitive Denoising Methods with Similar Performances

- ▷ **BM3D**: Wiener/DCT, non local self-similarity, patch clustering (Dabov, 2007)
- ▷ **NL-Bayes**: Bayesian estimation, clustering and Gaussian prior (Lebrun, 2013)
- ▷ **EPLL**: MAP estimation, Gaussian mixture prior (Zoran, 2011)
- ▷ **LSSC**: Non-local means, sparse coding (Mairal, 09)
- ▷ S-PLE (Wang, 2013), SOP (Ram, 2013), PLOW (Chatterjee, 2012), . . . , SKR (Takeda, 2007), SAFIR (Kervrann, 2006)

. . . are inspired from patch-based methods presented in 2005-2006:

- ▷ **NL-means**: Non-local means, image self-similarity (Buades, 2005)
- ▷ **UINTA**: Information theory, neighborhood entropy minimization (Awate, 2005)
- ▷ **FoE**: MRF, patch-based potential learning (Roth, 2005)
- ▷ **K-SVD**: sparse representation, over-complete dictionaries (Elad, 2006)

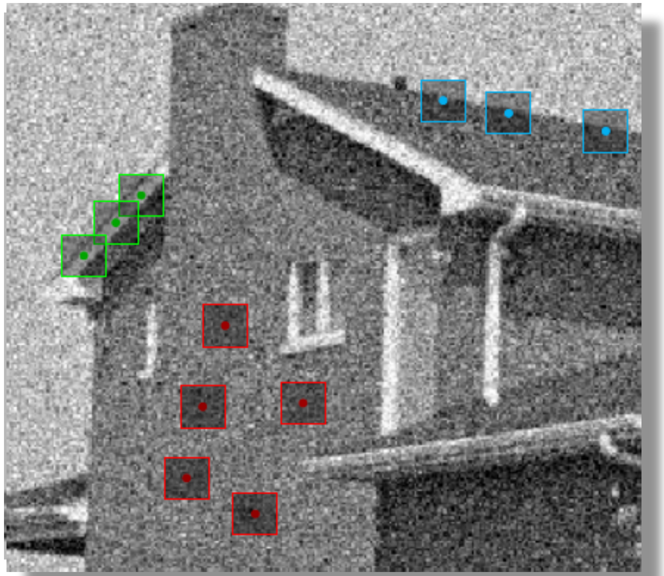
Overview

- ▷ We show that **"weakly" denoised versions** of the input image can serve to compute a performant patch-based aggregated estimator.
- ▷ We evaluate the **performance** of each patch estimator to compute the Exponentially Weighted Aggregation (EWA) (Leung & Barron, 2006) (Dalayan & Tsybakov, 2008) (Salmon & Le Pennec, 2009).
- ▷ The aggregation method is flexible enough to combine any standard denoising algorithms and has an interpretation with **Gibbs distribution**.
- ▷ PEWA is based on a **MCMC sampling** and is able to produce results that are comparable to the current state-of-the-art.

PEWA: A statistical **aggregation method** which combines **denoised image patches**, generalizes the **NL-means** and produces **state-of-the-art results**

Similar ideas: *SOS Boosting (Romano, 2015), Boosting "ShotGun" (Pierazzo, 2013), SAIF (Talebi, 2012)*

Image Patch Model and Estimator



▷ **Notations (patches):**

$\boldsymbol{f}(x)$: n-dimensional **unknown patch** at location $x \in \mathcal{X} \subset \mathbb{R}^2$

$\boldsymbol{v}(x)$: n-dimensional **noisy** patch (Gaussian noise)

$$\boldsymbol{v}(x) = \boldsymbol{f}(x) + \boldsymbol{\varepsilon}(x) \quad \text{with } \boldsymbol{\varepsilon}(x) \sim \mathcal{N}(0, \sigma^2 \mathbf{I}_{n\times n})$$

$\widehat{\boldsymbol{f}}(x)$: n-dimensional **patch estimator**

▷ **Empirical statistic (measure): detection of deviation from noise**

$$R(\widehat{\boldsymbol{f}}(x)) = \|\boldsymbol{v}(x) - \widehat{\boldsymbol{f}}(x)\|_n^2 - n\sigma^2$$

with this choice, we have $\mathbb{E}[R(\widehat{\boldsymbol{f}}(x))] = \mathbb{E}[\|\boldsymbol{f}(x) - \widehat{\boldsymbol{f}}(x)\|_n^2]$ (L_2 risk)

Aggregation by Exponential Weights

- Assume a set $\{\boldsymbol{f}_\lambda(x), \lambda \in \Lambda = \{1, \cdots, M\}\}$ of **pre-computed estimators**. We consider an aggregate that is the **weighted average of estimators** with some data-dependent weights:

$$\boxed{\widehat{\boldsymbol{f}}(x) = \sum_{\lambda=1}^{M} w_\lambda(x)\, \boldsymbol{f}_\lambda(x) \quad \text{such that} \quad w_\lambda(x) \geq 0 \quad \text{and} \quad \sum_{\lambda=1}^{M} w_\lambda(x) = 1.}$$

- We associate two probability measures $\boldsymbol{w}(x) = \{w_\lambda(x)\}$ and $\boldsymbol{\pi}(x) = \{\pi_\lambda(x)\}$, $\lambda \in \Lambda$ and we define the **Kullback-Leibler divergence** as (Rigollet, 2012):

$$D_{KL}(\boldsymbol{w}(x), \boldsymbol{\pi}(x)) = \sum_{\lambda=1}^{M} w_\lambda(x) \log\left(\frac{w_\lambda(x)}{\pi_\lambda(x)}\right).$$

The role of the distribution $\boldsymbol{\pi}$ is to put **a prior weight** on the estimators in the set.

Aggregation by Exponential Weights:
an Optimization Problem

▷ The **weights** are solutions of the optimization problem:

$$\widehat{\boldsymbol{w}}(x) = \arg \min_{\boldsymbol{w}(x) \in \mathbb{R}^M} \left\{ \sum_{\lambda=1}^{M} w_\lambda(x) \phi(R(\boldsymbol{f}_\lambda(x))) + \beta \, D_{KL}(\boldsymbol{w}(x), \boldsymbol{\pi}(x)) \right.$$
$$\left. - \alpha \left(\sum_{\lambda=1}^{M} w_\lambda(x) - 1 \right) - \sum_{\lambda=1}^{M} b_\lambda(x) w_\lambda(x) \right\}$$

where $\alpha, \beta > 0$, $b_\lambda(x) w_\lambda(x) = 0$ and $\phi : \mathbb{R} \to \mathbb{R}^+$.

▷ From the **Karush-Kuhn-Tucker conditions**, we get

$$\boxed{\widehat{w}_\lambda(x) \;=\; \frac{\exp(-\phi(R(\boldsymbol{f}_\lambda(x)))/\beta) \, \pi_\lambda(x)}{\sum_{\lambda'=1}^{M} \exp(-\phi(R(\boldsymbol{f}_{\lambda'}(x)))/\beta) \, \pi_{\lambda'}(x)},}$$

and β can be interpreted as a **“temperature” parameter**.

PEWA: Patch-based EWA estimator

- ▷ Assume $\{u_1, \cdots, u_L\}$ **"weakly" denoised versions** of v (Gaussian, Wiener, DCT, Wavelet, Median, Bilateral ...).
- ▷ An estimator $\boldsymbol{f}_\lambda(x)$ is a n-dimensional patch (denoted $\boldsymbol{u}_l(y)$) taken in $u_\ell, \ell \in \{1, \cdots, L\}$ at any location $y \in \mathcal{X}$.
- ▷ Our estimator is of the following form:

$$\boxed{\widehat{\boldsymbol{f}}(x) \;=\; \frac{1}{Z(x)} \sum_{\ell=1}^{L} \sum_{y \in \mathcal{X}} e^{-|R(\boldsymbol{u}_\ell(y))|/\beta}\, \pi_\ell(y)\, \boldsymbol{u}_\ell(y)}$$

where $Z(x)$ is a normalization constant and $\phi(z) = |z|$ favors estimators with **small deviations from noise**.

Patch-based EWA Estimator:
Gibbs Model and "Neighborhood" Prior

The patch-based EWA (PEWA) estimator is written in terms of **Gibbs distributions** as ($\beta = 4\sigma^2$, Leung, 2006):

$$
\begin{aligned}
\widehat{\boldsymbol{f}}_{\text{PEWA}}(x) &= \frac{1}{Z(x)} \sum_{\ell=1}^{L} \sum_{y \in \mathcal{X}} e^{-E(\boldsymbol{u}_\ell(y))}\, \boldsymbol{u}_\ell(y) \\
Z(x) &= \sum_{\ell'=1}^{L} \sum_{y' \in \mathcal{X}} e^{-E(\boldsymbol{u}_{\ell'}(y'))}, \\
E(\boldsymbol{u}_\ell(y)) &= \frac{\left| \|\boldsymbol{v}(x) - \boldsymbol{u}_\ell(y)\|_n^2 - n\sigma^2 \right|}{4\sigma^2} + \boxed{\frac{\|x - y\|_2^2}{2\tau^2}}.
\end{aligned}
$$

- ▷ **Prior:** favors patches located in the **spatial neighborhood** of x.
- ▷ **Monte-Carlo sampling:** method to approximately compute PEWA when the number of patch estimators is large.

Patch-based EWA Estimator:
Generalization of Non-Local means

PEWA is equivalent to NL-means if we choose $L = 1$, $u_{\ell=1} = v$, $\phi(z) = z$ and $\tau \to \infty$ ($\approx$ flat prior):

$$
\begin{aligned}
\widehat{\boldsymbol{f}}_{\scriptscriptstyle NLM}(x) &= \frac{1}{Z(x)} \sum_{y \in \mathcal{X}} e^{-E(\boldsymbol{v}(y))}\, \boldsymbol{v}(y), \qquad Z(x) = \sum_{y' \in \mathcal{X}} e^{-E(\boldsymbol{v}(y'))} \\
E(\boldsymbol{v}(y)) &= \frac{\|\boldsymbol{v}(x) - \boldsymbol{v}(y)\|_n^2 - n\sigma^2}{\beta} + \frac{\|x - y\|_2^2}{2\tau^2} \\
&\approx \frac{\|\boldsymbol{v}(x) - \boldsymbol{v}(y)\|_n^2}{\beta} + cte
\end{aligned}
$$

Non-local means (practice): selection of patches in a **fixed-size search window** (21×21 pixels).

"Data-driven" Monte-Carlo Sampling:
Computational Issues

Assume a random process $(\boldsymbol{F}_m(x))_{m\geq 0}$ and an initial noisy patch $\boldsymbol{F}_0(x) = \boldsymbol{v}(x)$. The **"data-driven"** MCMC procedure is based on the **Metropolis-Hastings algorithm**:

Draw a patch by considering a two-stage drawing procedure:

1. *draw uniformly a value ℓ in the set $\{1, 2, \cdots, L\}$.*
2. *draw a pixel $y = y_c + \gamma, y \in \mathcal{X}$, with $\gamma \sim \mathcal{N}(0, I_{2\times 2}\tau^2)$ and y_c is the position of the current patch. At the initialization $y_c = x$.*

Define $\boldsymbol{F}_{m+1}(x) = \begin{cases} \boldsymbol{u}_\ell(y) & \text{if } \alpha \sim U[0,1] \leq e^{-(E(\boldsymbol{u}_\ell(y)) - E(\boldsymbol{F}_m(x)))} \\ \boldsymbol{F}_m(x) & \text{otherwise.} \end{cases}$

“Data-driven” Monte-Carlo Sampling (contd’)
Computational Issues

If we assume the Markov chain is ergodic, homogeneous, irreductible, reversible and stationary, for any $\boldsymbol{F}_0(x)$, we have almost surely

$$\boxed{\lim_{T \to +\infty} \frac{1}{T - T_b} \sum_{m=T_b}^{T} \boldsymbol{F}_m(x) \approx \widehat{\boldsymbol{f}}_{\text{PEWA}}(x)}$$

▷ **MCMC algorithm (one patch):**

- $T \approx 1000$ is the maximum number of samples.
- The first $T_b = 250$ samples are discarded (burn-in phase).

▷ **Global image denoising:**

- Patch overlapping: multiple estimates $\widehat{\boldsymbol{f}}_{\text{PEWA}}(x)$ at a given pixel x.
- Uniform averaging: ”fusion” of n independent Markov chains at each pixel.

Denoising: Additive White Gaussian Noise on Natural Images

Set of 25 images. Top left: images from the BM3D website (cs.tut.fi/˜foi/GCFBM3D/); Bottom left: images from IPOL (ipol.im); Right: images from the Berkeley segmentation database (eecs.berkeley.edu/Research/Projects/CS/ vision/bsds/).

Experimental Results:
Artificially Noisy Data

A **two-step procedure** with the parameters $\tau = 7, n = 7 \times 7$ and $L = 4$:

- ▷ $\underline{1^{st}}$ **iteration:** estimation using the noisy image v and 3 denoised images u_l (DCT shrinkage thresholds: $\{1.25; 1.50; 1.75\} \times \sigma$) (Yu, 2011).
- ▷ $\underline{2^{nd}}$ **iteration:** estimation as before using the 1^{st} PEWA estimator considered as an additional denoised image (improvement in the range of 0.2 to 0.5 dB).

	$\sigma = 5$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 25$	$\sigma = 50$	$\sigma = 100$
PEWA 1^{st} iteration	38.27	34.39	32.26	30.76	29.62	26.00	22.35
PEWA 2^{nd} iteration	38.54	**34.75**	**32.67**	**31.26**	**30.15**	**26.95**	23.76
BM3D [Dabov, 2007]	**38.64**	**34.78**	**32.68**	**31.25**	**30.19**	**26.97**	**24.08**
NL-Bayes [Lebrun, 2013]	**38.60**	**34.75**	32.48	**31.22**	30.12	26.90	23.65
S-PLE [Wang, 2013]	38.17	34.38	32.35	30.67	29.77	26.46	23.21
NL-means [Buades, 2005]	37.44	33.35	31.00	30.16	28.96	25.53	22.29
DCT [Yu, 2011]	37.81	33.57	31.87	29.95	28.97	25.91	23.08

Table 1: Average of denoising results over 25 tested images for several values of σ (white Gaussian noise). The experiments with NL-Bayes, S-PLE, NL-means and DCT have been performed using the implementations of IPOL (www.ipol.im).

Experimental Results (artificial data)

	$\sigma = 5$	$\sigma = 10$	$\sigma = 15$	$\sigma = 20$	$\sigma = 25$	$\sigma = 50$	$\sigma = 100$
Cameraman	38.20	34.23	31.98	30.60	29.48	26.25	22.81
Peppers	38.00	34.68	32.75	31.40	30.30	26.69	22.84
House	39.56	36.40	34.86	33.72	32.77	29.29	25.35
Lena	38.57	35.78	34.12	32.90	31.89	28.83	25.65
Barbara	38.09	34.73	32.86	31.43	30.28	26.58	22.95
Boat	37.12	33.75	31.94	30.64	29.65	26.64	23.63
Man	37.68	33.93	31.93	30.50	29.50	26.67	24.15
Couple	37.35	33.91	31.98	30.57	29.48	26.02	23.27
Hill	37.01	33.52	31.69	30.50	29.56	26.92	24.49
Alley	36.29	32.20	29.98	28.54	27.46	24.13	21.37
Computer	39.04	35.13	32.81	31.23	30.01	26.38	23.27
Dice	46.82	43.87	42.05	40.58	39.36	35.33	30.82
Flowers	43.48	39.67	37.47	35.90	34.55	30.81	27.53
Girl	43.95	41.22	39.52	38.27	37.33	34.14	30.50
Traffic	37.85	33.54	31.13	29.58	28.48	25.50	22.90
Trees	34.88	29.93	27.49	25.86	24.69	21.78	20.03
Valldemossa	36.65	31.79	29.25	27.59	26.37	23.18	20.71
Aircraft	37.59	34.62	33.00	31.75	30.72	27.68	24.99
Asia	38.67	34.46	32.25	30.73	29.60	26.63	24.32
Castle	38.06	34.13	32.02	30.56	29.49	26.15	23.09
Man Picture	37.78	33.58	31.27	29.73	28.44	24.65	21.50
Maya	34.72	29.64	27.17	25.42	24.28	22.85	18.17
Panther	38.53	33.91	31.56	30.02	28.83	25.59	22.75
Tiger	36.92	32.85	30.63	29.13	27.99	24.63	21.90
Young man	40.79	37.36	35.58	34.30	33.25	29.59	25.20
Average	**38.54**	**34.75**	**32.67**	**31.26**	**30.15**	**26.95**	**23.76**

Table 2: PSNR values are averaged over 3 different noise realizations.

Experimental Results

Image	Peppers (256 × 256)				House (256 × 256)				Lena (512 × 512)				Barbara (512 × 512)			
σ	5.00	15.00	25.00	50.00	5.00	15.00	25.00	50.00	5.00	15.00	25.00	50.00	5.00	15.00	25.00	50.00
PEWA 1 (W) (5×5)	36.69	30.58	27.50	22.85	37.89	31.88	28.55	23.49	37.27	31.43	28.30	23.45	36.39	30.18	29.31	22.71
PEWA 2 (W) (5×5)	37.45	32.20	29.72	26.09	38.98	34.27	32.13	28.35	38.05	33.40	31.11	27.80	37.13	31.94	29.47	25.58
PEWA 1 (W) (7×7)	36.72	30.60	27.60	22.82	37.90	31.90	28.59	23.52	37.26	31.45	28.33	23.45	36.40	30.18	27.32	22.71
PEWA 2 (W) (7×7)	37.34	32.34	30.11	26.53	39.00	34.57	32.51	29.04	38.00	33.65	31.56	28.40	37.00	32.10	30.00	26.20
PEWA 1 (D) (5×5)	37.70	32.45	29.83	26.01	39.28	34.23	31.79	27.72	38.46	33.72	31.33	27.59	37.71	32.20	29.55	25.58
PEWA 2 (D) (5×5)	37.95	**32.80**	30.20	**26.66**	39.46	34.74	31.67	29.15	**38.57**	33.96	31.81	28.43	38.03	32.70	30.03	26.01
PEWA 1 (D) (7×7)	37.71	32.43	29.87	26.00	39.27	34.26	31.79	27.71	38.45	33.72	31.25	27.62	37.70	32.30	29.84	26.20
PEWA 2 (D) (7×7)	38.00	32.75	30.30	26.69	39.56	34.83	32.77	29.29	38.58	34.12	31.89	28.83	38.09	32.86	30.28	26.58
PEWA Basic (7×7)	36.88	31.34	29.47	26.02	37.88	34.13	32.14	28.25	37.39	33.26	31.20	27.92	36.80	31.89	29.76	25.83
NL-means (7×7)	36.77	30.93	28.76	24.24	37.75	32.36	31.11	27.54	36.65	32.00	30.45	27.32	36.79	30.65	28.99	25.63
BM3D	38.12	32.70	30.16	26.68	39.83	34.94	32.86	29.69	38.72	34.27	32.08	29.05	38.31	33.11	30.72	27.23

Table 3: Comparison of several versions of PEWA ((W)iener), (D)CT, Basic) and BM3D on a few standard images corrupted with white Gaussian noise.

Experimental Results:
Artificially Noisy "Lena" Image

MCMC-based PEWA
PSNR = 31.58 db / Timings = 12 s

"Exact" PEWA
PSNR = 31.85 db / Timings = 4 min 53 s

Figure 1: "Lena" image corrupted with white Gaussian noise ($\sigma = 20$). Left: MCMC-based PEWA (1000 samples) applied to 7×7 non-overlapping patches. Right: "exact" PEWA applied to 7×7 non-overlapping patches and inspection of all image patches in the image ($L \times |\mathcal{X}|$ patches).

Experimental Results:
Artificially Noisy “Barbara” Image

MCMC-based PEWA
PSNR = 29.58 db / Timings = 12 s

”Exact” PEWA
PSNR = 29.84 db / Timings = 4 min 45 s

Figure 2: ”Barabra” image corrupted with white Gaussian noise ($\sigma = 20$). Left: MCMC-based PEWA (1000 samples) applied to 7×7 non-overlapping patches. Right: ”exact” PEWA applied to 7×7 non-overlapping patches and inspection of all image patches in the image ($L \times |\mathcal{X}|$ patches).

Experimental Results:
Artificially Noisy “Barbara” Image

MCMC-based PEWA	"Exact" PEWA
PSNR $=$ 29.58 db / Timings $=$ 12 s	PSNR $=$ 29.84 db / Timings $=$ 4 min 45 s

"Cameraman" image corrupted with white Gaussian noise ($\sigma = 20$). Full image denoising:

- Exact PEWA: 25 min and 32 sec ($\beta = 10., \phi(z) = |z|$ and 5×5 patches): PSNR $=$ 30.22.
- MCMC-based PEWA: 32 sec ($\beta = 10., \phi(z) = |z|$ and 5×5 patches): PSNR $=$ 30.28.

Experimental Results:

Artificially Noisy “Cameraman” Image

Cameraman ($\sigma = 20$)	$\phi(z) = \exp(z)$	$\phi(z) = z^2$	$\phi(z) = z$	$\phi(z) = \|z\|$	$\phi(z) = H_4(z)$	$\phi(z) = \sqrt{z}$	$\phi(z) = (z)_+$
$\beta = 0.1$	29.56 30.11	29.57 30.11	25.09 26.11	29.53 30.11	28.83 29.11	29.66 30.27	27.89 29.00
$\beta = 0.5$	29.58 30.17	29.57 30.18	25.23 26.43	29.61 30.22	29.39 29.76	30.07 30.36	27.78 28.78
$\beta = 1.0$	29.60 30.20	29.61 30.20	25.36 26.69	29.66 30.28	29.66 30.16	30.19 30.19	27.88 29.00
$\beta = 2.0$	29.64 30.26	29.68 30.28	25.58 27.12	29.80 30.39	29.96 30.44	30.18 29.95	28.17 29.38
$\beta = 3.0$	29.69 30.30	29.73 30.32	25.83 27.56	29.92 30.42	30.08 30.47	30.11 29.75	28.50 29.68
$\beta = 3.5$	29.72 30.33	29.79 30.37	25.99 27.80	29.98 30.46	30.14 30.47	30.06 29.65	28.66 29.80
$\beta = 4.0$	29.75 30.35	29.81 30.37	26.18 28.04	30.04 30.47	30.17 30.46	30.01 29.59	28.81 29.93
$\beta = 4.5$	29.76 30.37	29.83 30.39	26.39 28.29	30.08 30.46	30.20 30.45	29.97 29.51	28.95 30.02
$\beta = 5.0$	29.78 30.36	29.90 30.42	26.61 28.50	30.12 30.47	30.23 30.41	29.97 29.51	29.09 30.11
$\beta = 6.0$	29.84 30.37	29.95 30.45	27.09 28.97	30.18 30.42	30.27 30.34	29.83 29.29	29.33 30.21
$\beta = 7.0$	29.88 30.39	30.00 30.46	27.54 29.30	30.23 30.37	30.28 30.28	29.74 29.20	29.54 30.28
$\beta = 8.0$	29.93 30.42	30.05 30.46	27.99 29.59	30.27 30.35	30.28 30.28	29.66 29.04	29.69 30.32
$\beta = 9.0$	29.99 30.44	30.09 30.46	28.37 29.80	30.28 30.28	30.29 30.14	29.58 28.95	29.83 30.33
$\beta = 10.0$	30.00 30.44	30.09 30.46	28.67 29.95	30.29 30.25	30.28 30.05	29.51 28.82	29.94 30.30
$\beta = 15.0$	30.16 30.43	30.23 30.42	29.69 30.17	30.26 30.03	30.09 29.74	29.19 28.38	30.19 30.18
$\beta = 20.0$	30.26 30.38	30.29 30.30	30.05 30.10	30.18 29.83	29.86 29.43	28.90 28.01	30.20 30.00
$\beta = 30.0$	30.31 30.20	30.27 30.06	30.10 29.78	29.94 29.53	29.44 28.91	28.47 27.48	30.05 29.67
$\beta = 40.0$	30.24 30.00	30.13 29.81	29.96 29.54	29.72 29.23	29.09 28.48	28.18 27.10	29.85 29.39

Figure 2: "Barabra" image corrupted with white Gaussian noise ($\sigma = 20$). Left: MCMC-based PEWA (1000 samples) applied to 7×7 non-overlapping patches. Right: "exact" PEWA applied to 7×7 non-overlapping patches and inspection of all image patches in the image ($L \times |\mathcal{X}|$ patches).

Experimental Results:
Artificially Noisy Data

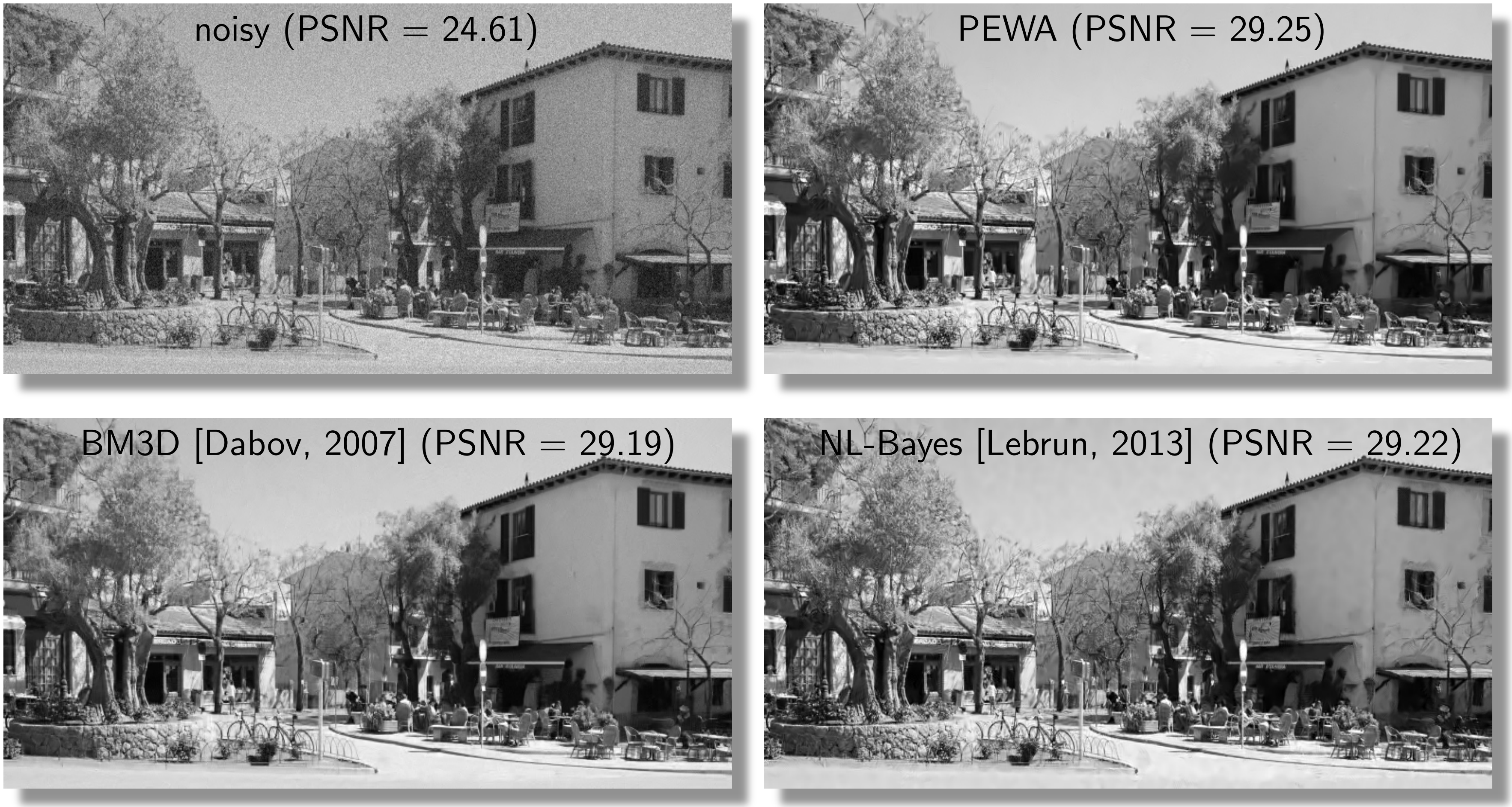


Figure 3: "Valldemossa" image corrupted with white Gaussian noise ($\sigma = 15$). The PSNR values of the 3 images denoised with DCT-based transform (Yu, 2011) and combined with PEWA are 27.78, 27.04 and 26.26.

Experimental Results
Artificially Noisy Data

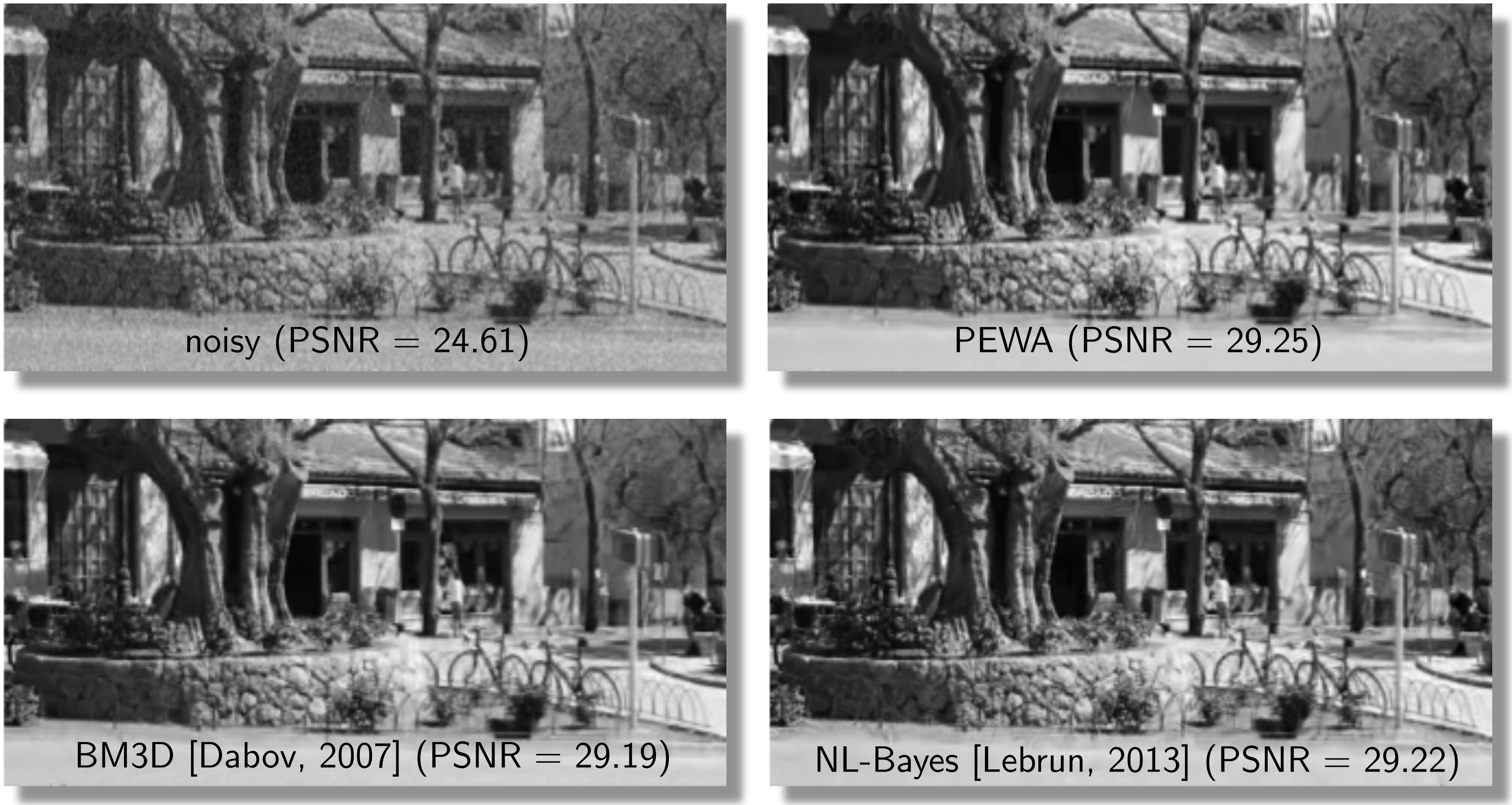


Figure 4: "Valldemossa" image corrupted with white Gaussian noise ($\sigma = 15$). The PSNR values of the 3 images denoised with DCT-based transform [Yu, 2011] and combined with PEWA are 27.78, 27.04 and 26.26.

Experimental Results (artificial data)

Figure 5: Castle image corrupted with white Gaussian noise ($\sigma = 25$). The PSNR values of the 3 images denoised with DCT-based transform (Yu, 2011) and combined with PEWA are 25.77, 24.26 and 22.85.

Experimental Results

Artificially Noisy Data

noisy	PEWA	BM3D [Dabov, 2007]	NL-Bayes [Lebrun, 2013]
(PSNR = 20.18)	(PSNR = 29.49)	(PSNR = 29.36)	(PSNR = 29.48)

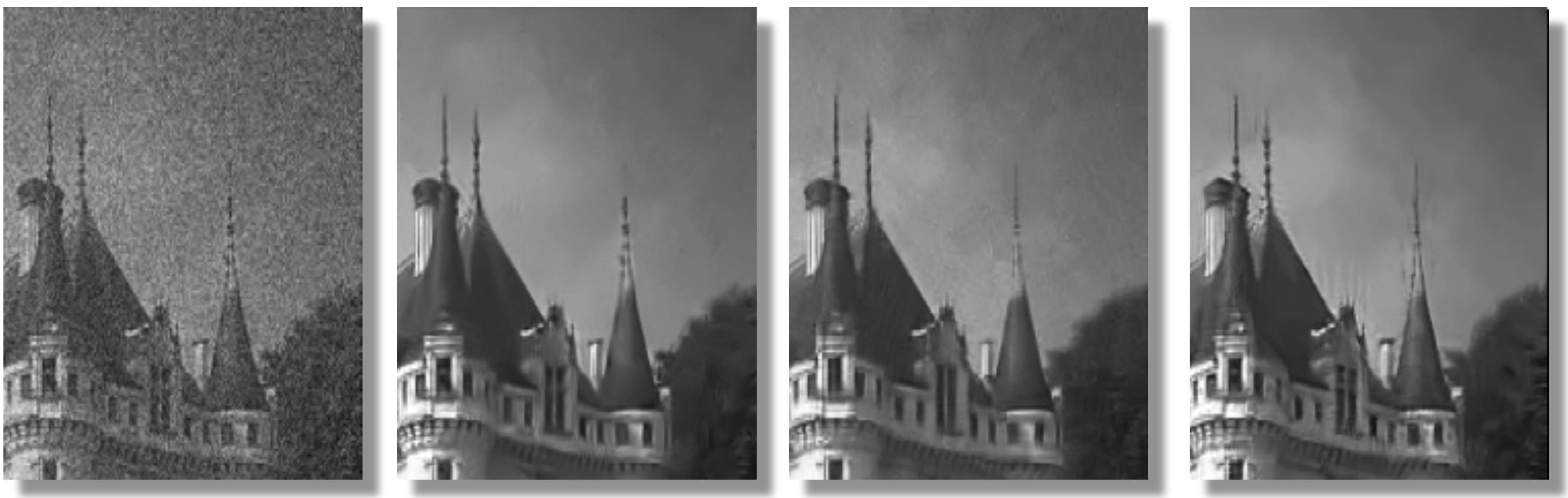
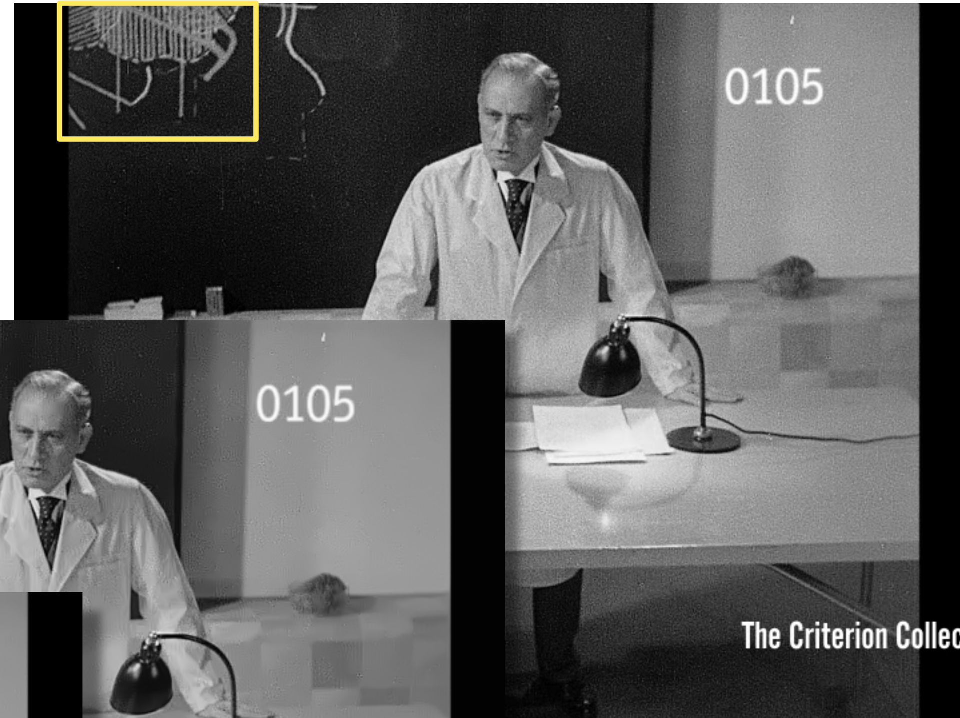


Figure 6: Castle image corrupted with white Gaussian noise ($\sigma = 25$). The PSNR values of the 3 images denoised with DCT-based transform (Yu, 2011) and combined with PEWA are 25.77, 24.26 and 22.85.

Denoising of Real Old Pictures



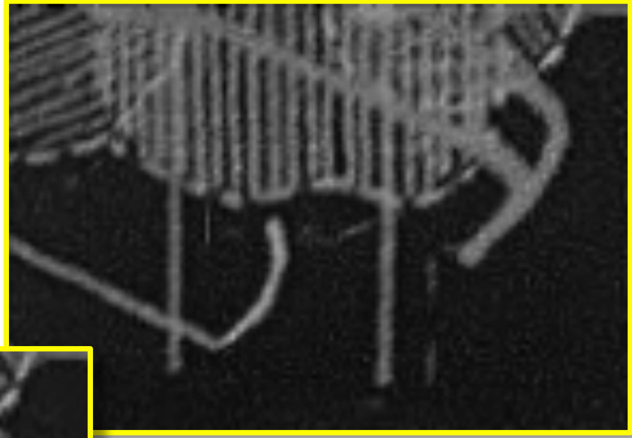
Gaussian noise and spatially-varying variance:

$$v(x) = f(x) + \varepsilon(x), \varepsilon(x) \sim \mathcal{N}(0, \sigma^2(x))$$

Empiricial statistic:

$$R(\widehat{\boldsymbol{f}}(x)) = \|\boldsymbol{v}(x) - \widehat{\boldsymbol{f}}(x)\|_n^2 - \boxed{n\sigma^2(x)}$$

Denoising of Real Old Pictures



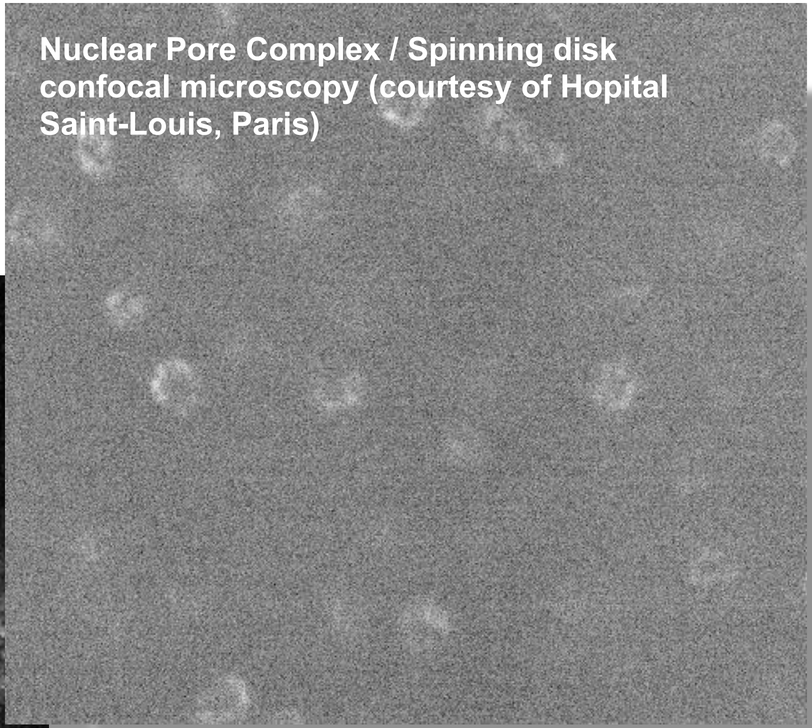
Gaussian noise and spatially-varying variance:

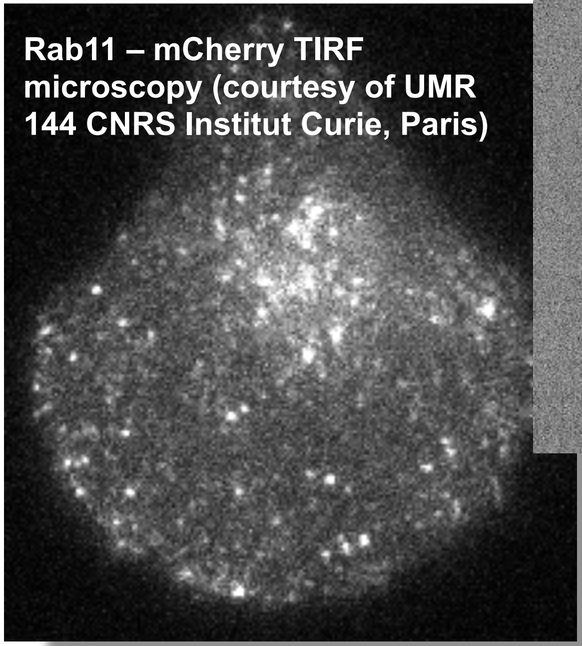
$$v(x) = f(x) + \varepsilon(x), \varepsilon(x) \sim \mathcal{N}(0, \sigma^2(x))$$

Empiricial statistic:

$$R(\widehat{\boldsymbol{f}}(x)) = \|\boldsymbol{v}(x) - \widehat{\boldsymbol{f}}(x)\|_n^2 - \boxed{n\sigma^2(x)}$$

Denoising in Fluorescence Microscopy to Preserve Cell Integrity





Poisson-Gaussian Noise in Fluorescence Microscopy

$$\boxed{v(x) = g_0 \aleph(x) + \epsilon(x)}$$

- $\triangleright$ $v(x)$ is the intensity observed at space-time location $x \subset \mathbb{R}^d$.
- $\triangleright$ g_0 is gain of the overall electronic system.
- $\triangleright$ $\aleph(x)$ is the number of photo-electrons at pixel x assumed to be Poisson distributed with unknown mean $\theta(x)$.
- $\triangleright$ $\epsilon(x) \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is a white Gaussian noise and represents "dark current".

Denoising in Fluorescence Microscopy
(single micro-patterned cell)

$$\mathrm{Var}[v(x)] = g_0 \mathbb{E}[v(x)] + \sigma_\epsilon^2$$

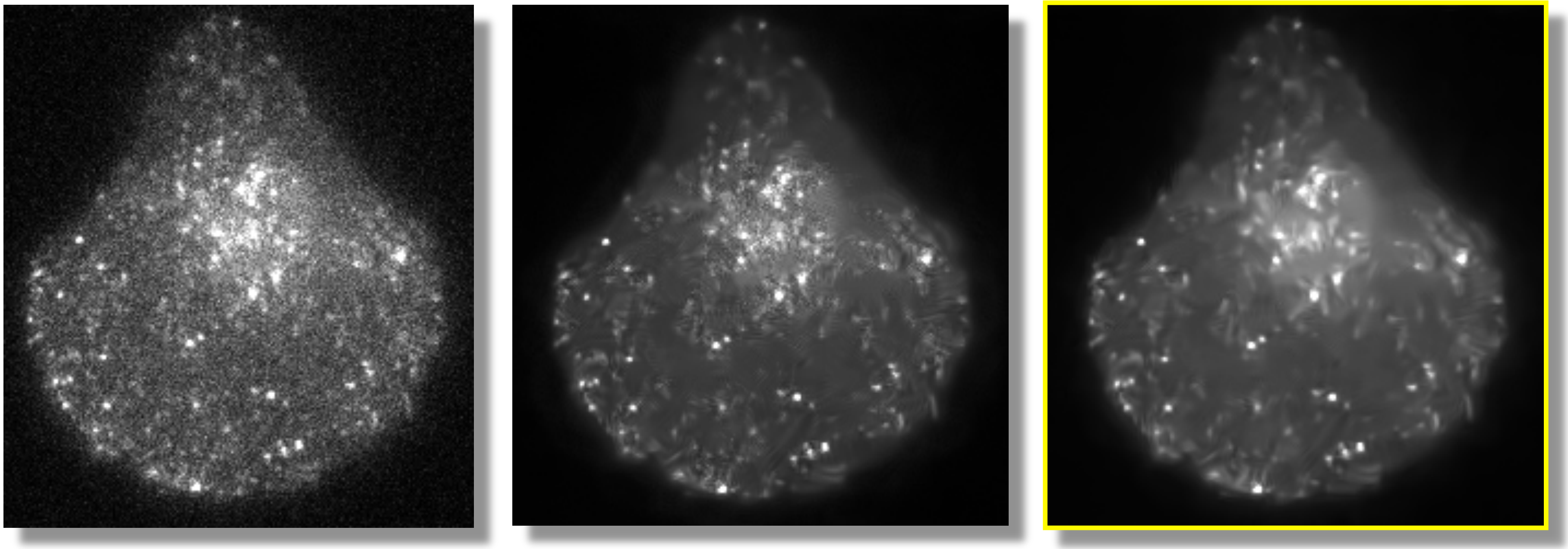


Figure 9: Rab11-mCherry protein observed in TIRF microscopy (1 px=80-100 nm). Left: noisy image ; middle: PEWA (Gaussian) ; right: PEWA (Poisson-Gaussian).

$$\begin{aligned} R(\widehat{\boldsymbol{f}}(x)) &= \|\boldsymbol{v}(x) - \widehat{\boldsymbol{f}}(x)\|_n^2 - g_0\langle \mathbf{1}, \boldsymbol{v}(x)\rangle - n\sigma_\epsilon^2 \\ \beta &= 4(n^{-1} g_0 \langle \mathbf{1}, \boldsymbol{v}(x)\rangle + \sigma_\epsilon^2) \end{aligned}$$

Denoising in Fluorescence Microscopy
(3D data / single cell)

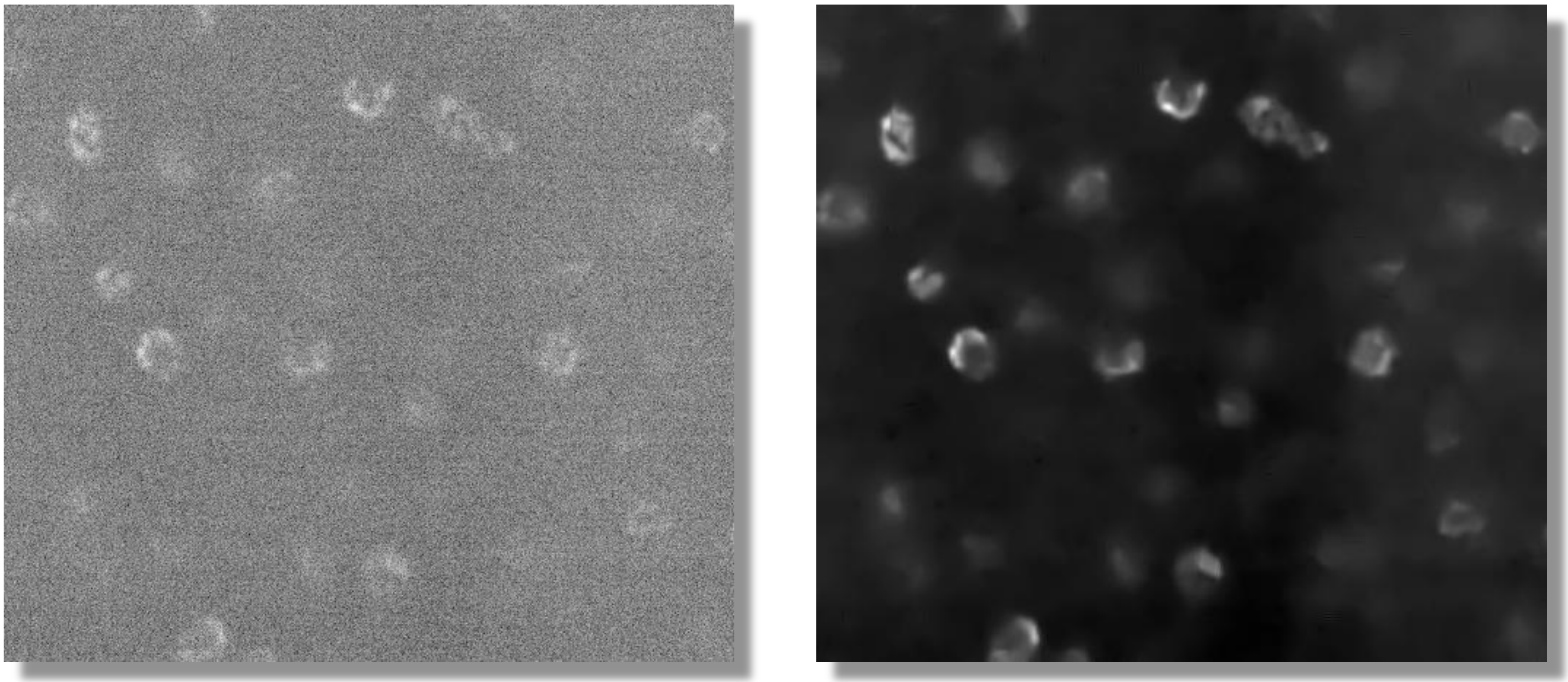


Figure 10: Nuclear Pore Complex (spinning disk confocal / 1 px=80-100 nm). Left: noisy image sequence; right: PEWA.

Summary

A statistical **aggregation method** which combines **denoised image patches** and generalizes the **NL-means** method:

Good news:

- ▷ Easy to use / intuitive parameters
- ▷ Applicable to a wide range of parametric noise models
- ▷ Flexible: "cocktail" of heterogeneous basic denoising algorithm
- ▷ Comparable to BM3D
  (e.g. 1 iteration: 0.2 db lower than BM3D)

Bad news:

- ▷ Comparable to BM3D . . . but not more
- ▷ Combination of sophisticated methods (BM3D, NL-Bayes) is not better
- ▷ Timings: 1-2 min for 512×512 image (basic implementation)

Related works

Buades A., Coll B. & Morel J.-M. (2005) *SIAM J. MMS*, **4**(2):490-530.

Dabov K., Foi A., Katkovnik V. & Egiazarian K. (2007) *IEEE T. Image Processing* **16**(8):2080-2095.

Dalayan A.S. & Tsybakov A. B. (2008) *Machine Learning* **72**:39-61.

Leung G. & Barron A. R. (2006) *IEEE T. Information Theory*, **52**:3396-3410.

Lebrun M., Buades A. & Morel J.-M. (2013) *Image Processing On Line* **3**:1-12

Le Montagner Y., Angelini E., Olivo-Marin J.C. (2014) *IEEE T. Image Processing* **23**(3):1255-1268.

Louchet C. & Moisan L. (2013) *SIAM J. Imaging Science* **6**(4):2640-2684.
(Louchet, C. & Moisan, L., In Proc. EUSIPCO 2008)

Rigollet P. & Tsybakov A. B. (2012) *Statistical Science* **27**(4):558-575.

Salmon J. & Le Pennec E. (2009) In *Proc. ICIP'09*, pp. 2977-2980, Cairo, Egypt.

Yu G. & Sapiro G. (2011) Image Processing On Line (http://dx.doi.org/ 10.5201/ipol.2011.ys-dct).

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Kervrann, C. (2014) PEWA: Patch-based Exponentially Weighted Aggregation for image denoising. In *Proc. Neural Information Processing Systems (NIPS'14)*, Montreal, Canada.

Boulanger, J., Kervrann, C., Bouthemy, P., Elbau, P., Sibarita, J.-B., and Salamero, J. (2010). *IEEE T. Medical Imaging*, 29(2):442– 453.

Thank you !